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NCSX experiment terminated

Under Secretary for Science of the U. S. Department of Energy (DOE), Raymond Orbach issues the following statement on May 23:

Future of the Princeton Plasma Physics Laboratory (PPPL)

"In late 2006, it became clear that National Compact Stellarator Experiment (NCSX) construction project would not be able to meet its approved baseline total project cost of $102M or its completion date of July 2009. Since then, DOE, Princeton University, and PPPL have worked extensively together to understand the issues and plot a course of action that maximizes the benefits for the scientific community and the taxpayers, and ensures an exciting path for PPPL research well into the future. Following several internal and external reviews over the past 18 months, it has been concluded that the budget increases, schedule delays and continuing uncertainties of the NCSX construction project necessitate its closure, and that PPPL's future as a world-leading center of fusion energy and plasma sciences is more assured by a renewed focus on the successful Spherical Torus confinement concept.

"The Office of Science always weighs the scientific benefits to be obtained from facilities against the cost to the taxpayer — in this case the escalating costs and remaining uncertainties make continuation of the construction project untenable. The latest cost estimate is $170M with an August 2013 scheduled completion. An Office of Science review (April 2008) concluded that the project has not yet met the requirements needed to approve a new baseline cost and schedule. This puts the future of research at PPPL in unnecessary peril, and increases the burden on the DOE fusion energy sciences program. It would require the premature closure of the Spherical Torus experiment (NSTX), a proven, productive, world-leading scientific facility, while creating an uncertain gap in research capabilities at PPPL.

"This would result in a loss of opportunities for a large number of collaborators in the research community and constrain the ability to start new initiatives during the ITER era.

"The highest priority of the U.S. fusion program is participation in the International ITER burning plasma experiment, which is based on the tokamak concept. The Spherical Torus is closely related to the tokamak, and experiments planned for the next several years in the NSTX facility promise many exciting discoveries that should directly impact our ability to understand the new plasma regimes expected in ITER. The Spherical Torus may also prove to be a prototype for the next step for the U.S. domestic fusion program. Planned upgrades for the Spherical Torus experiment at PPPL can keep this facility

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at the forefront of fusion science research in the world well into the future. As such, a concentration on the Spherical Torus better positions PPPL to remain a center of excellence for fusion energy and plasma sciences, and thereby compete for new areas of leadership in the future fusion program.

"Closure of the Compact Stellarator construction effort will be managed to capture many benefits of the project. PPPL will complete the special modular and toroidal field coils in FY 2008. A modest engineering effort will document the R&D achievements to date, and continue to retire remaining risks of the Compact Stellarator design to allow revisiting this particular design if future developments in the fusion program warrant it. In addition, the U.S. fusion program will increase its investments in theory and smaller, focused experiments on stellarator concepts to maintain its interest in future development of these exciting plasma confinement concepts. We believe this decision is in the best interests of the American fusion program PPPL and Princeton University. Our decision reflects our strong commitment to the future of PPPL as a center of scientific excellence, including the prospect that it will compete successfully for opportunities to extend its work in plasma and fusion science in a number of important and promising new directions."

NCSX construction activities are now being closed following DOE's decision to terminate the project. All modular coils and toroidal field coils (18 of each) will be completed before the project closes. The modular coils have all been wound to the required ± 0.5 mm tolerance. The first pair of modular coils has been assembled, also within ± 0.5 mm tolerance, demonstrating the feasibility of the critical inter-coil joint design. The first two 3-coil subassemblies are nearly complete and are planned to be completed. All completed machine components, specialized tooling, and documentation will be secured and stored so that it would be possible to complete construction in the future should circumstances warrant. We will carefully document what was learned on NCSX and the engineering solutions that were developed.

The PPPL-ORNL team is grateful for the world stellarator community's support of NCSX throughout its history.

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TJ-II achieves new performance level

An important milestone was accomplished on 22 May 2008, in the TJ-II stellarator at the Fusion National Laboratory (CIEMAT) in Madrid. On the first day of operation with the second neutral beam injector, thanks to the substantial increase of available heating power and to the plasma density control provided by the lithium-coated first wall, TJ-II has achieved (domestic) record values of electron line density (7×10^{19} particles/m^3) and plasma energy content (5 kJ). This important achievement opens for TJ-II a new relevant scenario of high-density plasmas.

Figure 1 shows the newly extended operational range of plasma density and energy content (black stars in the upper right corner).

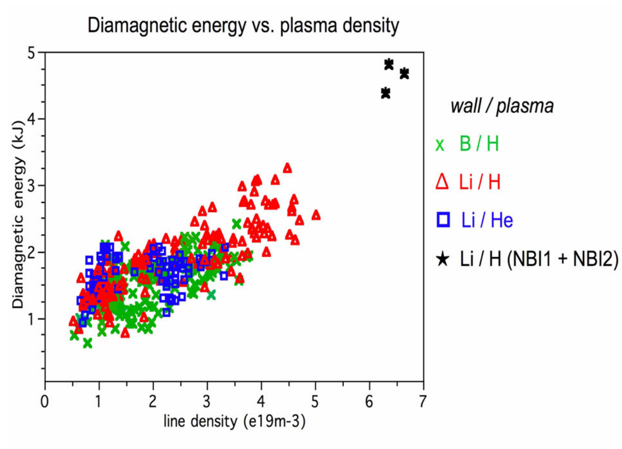

Fig. 1. Plasma energy vs line density in TJ-II.

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SIESTA: A Scalable Iterative Equilibrium Solver for Toroidal Applications

Magnetohydrodynamic (MHD) equilibria in toroidally confined plasmas are computed with various levels of approximation. In the simplest case, a geometrical symmetry of the plasma (such as toroidal symmetry in a tokamak) reduces the problem to a single equation (the Grad-Shafranov equation) for the low-dimensional (2-D) magnetic flux function, which nevertheless requires numerical solution. In 3-D geometry (rippled tokamaks, stellarators), the assumption of toroidally nested magnetic flux surfaces is sufficient to allow efficient numerical solution methods based on coordinate inverse methods [1]. However, the presence of magnetic islands can often lower the energy of 3-D configurations and cannot be described using inverse methods.

The SIESTA (Scalable Iterative Equilibrium Solver for Toroidal Applications) code described here builds upon the works of Chodura and Schlüter [2] and Harafuji et al. [3] to develop an iterative method for minimizing the total (magnetic and plasma) energy in 3-D plasmas:

$$W = \oint dV\left(\frac{1}{2\mu_0}B^2 + \frac{p}{\gamma - 1}\right) \tag{1}$$

The ideal MHD equations are used to obtain finite constrained variations of the magnetic field $\mathbf{B}$ and pressure p, corresponding to discrete versions of Faraday's law and density conservation, combined with ideal Ohm's law ($\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$) and adiabaticity ($p = n^{\gamma}$):

$$\delta\mathbf{B}(\xi) = \nabla \times (\xi \times \mathbf{B})$$

$$\delta p(\xi) = (\gamma - 1)\xi \bullet \nabla p - \gamma\nabla \bullet (p\xi) \tag{2}$$

The perturbed MHD vector displacement $\xi = \mathbf{v}\Delta t$ is treated as an independent 3-D variational parameter and can be used to find a stationary (local) minimum energy state corresponding to the ideal MHD force balance:

$$\mathbf{F}_{\mathrm{MHD}} \equiv \mathbf{J} \times \mathbf{B} - \nabla p = 0 \tag{3}$$

where $\mu_0\mathbf{J} = \nabla \times \mathbf{B}$ is the plasma current.

SIESTA departs significantly in a number of ways from the traditional iterative methods of finding a solution to Eq. (3). As discussed in more detail below, SIESTA solves a set of coupled differential equations for the displacement vectors and does not require following magnetic field lines at any point in the computation [3, 4]. This may be significant for accurately resolving magnetic fields in stochastic regions where the randomization of magnetic field lines leads to large spatial integration paths and computation times [5].

We now consider some of the specific features used in SIESTA. First, the 3-D inverse coordinates from the VMEC equilibrium code [1] are used as a background coordinate system [6] for the equilibrium computations. While the introduction of these nonorthogonal curvilinear coordinates complicates the finite difference equations, it has the appealing physical advantage of providing a nearby equilibrium state from which the displacements in Eq. (2) are expected to be small, even when the initial closed magnetic surface topology is broken by islands.

The second distinction of SIESTA is that it uses a Newton solver (based on the expected boundedness of the displacements) to rapidly converge to equilibrium (in typically tens of iterations). This contrasts to using a series of very small displacements (corresponding to a small time step in the momentum equation, not used here) to iterate Eq. (2) many (1000+) times to reach the equilibrium state in Eq. (3). The Newton iteration used in SIESTA is obtained from the field and pressure perturbations given by Eq. (2), which are used in Eq. (3) to update the variational displacements.
With $\mathbf{B}_{n+1} = \mathbf{B}_n + \delta\mathbf{B}(\xi)$, $p_{n+1} = p_n + \delta p(\xi)$:

$$\mathbf{F}_{n+1}(\xi) \equiv \tag{4}$$

$$\mathbf{F}_n + \delta\mathbf{J}(\xi) \times \mathbf{B}_n + \mathbf{J}_n \times \delta\mathbf{B}(\xi) - \nabla\delta p(\xi) + O(\xi^2) = 0$$

Here, $\mathbf{F}_n$ ($\mathbf{B}_n$, $\mathbf{J}_n$) is the MHD force (magnetic field, current) at the nth (previous) iteration step, and the second-order term in ξ is $\delta\mathbf{J} \times \delta\mathbf{B}$. If this term is ignored, then Eq. (4) can be converted to a linear matrix equation of the form:

$$H_{ij}x_j = f_j \tag{5}$$

where $\mathbf{H}$ is the Hessian matrix. Here, the x_i are composed of the radially discretized Fourier harmonics of the contravariant components of the displacement $\sqrt{g}\xi \bullet \nabla\alpha$, where $\alpha \in (s, u, v)$ represents the VMEC curvilinear flux (in SIESTA, $s = \sqrt{\Phi}$ is the "polar" radius) and angle coordinates (u,v).

A third unique feature of SIESTA is based on the observation that the highest-order differential operators in Eq. (4) are second-order in the radial coordinate. This implies that $\mathbf{H}$ in Eq. (5) is *block tri-diagonal*. It is possible to solve Eq. (5) *exactly* using a scalable and efficient algorithm. The algorithm avoids fill-in so that efficient storage is maintained during the solution of Eq. (5). This is important for future applications of SIESTA to analyze high temperature, ITER-regime plasma conditions which will require very fine radial resolutions to resolve islands at a large magnetic Reynolds number S.

Note that all relevant space scales of linearized ideal MHD — compressional and shear Alfvén, and sound wave scales — are included in the Hessian matrix **H** in Eq. (5). Therefore, the condition number of **H** can be quite large, especially at fine radial resolutions. The solution of Eq. (5) therefore requires an appropriate preconditioner to avoid numerical inaccuracies. Let **P** be a matrix that is invertible and for which $\mathbf{P}^{-1}\mathbf{H} \sim \mathbf{I}$. Such a preconditioner can be obtained in a number of ways, by adding (1) a small Levenberg-Marquardt diagonal element to **H** or (2) a small parallel velocity damping term $\sim \mu_{||}\xi_{||}$ to the linearized force, which eliminates the approximate null space of **H** for small wavelength displacements parallel to **B**. This latter method works well in general and has been used in the calculations described here. Note that the MHD force in Eq. (3) is not modified by $\mu_{||}$ since only the preconditioner includes this term for numerical stability.

SIESTA uses the reverse-communication GMRES package available from CERFACS [7] to solve Eq. (5) iteratively. Right-preconditioning provides the best convergence of the GMRES algorithm for our problem. Note that the importance of the preconditioner is that it coalesces the disparate Alfvénic and sonic scales thus making the condition number of the preconditioned Hessian close to unity. Typically 100 iterations of GMRES are sufficient to solve the linearized forces to a normalized squared force residual $\sim 10^{-20}$. A nonlinear iteration loop — n in Eq. (4) — is required to solve the MHD force equation Eq. (3), and requires tens of iterations to reduce the nonlinear force residual from an initial value of $\sim 10^{-5}$ (the interpolated nested VMEC solution) to $\sim 10^{-20}$.

When running SIESTA, a simple tri-diagonal solver is first used to reduce the initial force residuals arising from interpolating the VMEC equilibrium onto the SIESTA mesh. A set of resonant perturbations is applied at low-order rational surfaces to break the surfaces. In addition, operator splitting is used to apply a resistive perturbation $\delta B_{n+1} \sim -\nabla \times \eta J_n$ to the magnetic field perturbation for low iteration values, $n \leq 5$, in Eq. (4). This diffuses resonant current sheets, allowing islands to expand to decrease the magnetic energy in Eq. (1).

An equilibrium solution obtained by SIESTA for a D3D-like tokamak case with $\langle\beta\rangle \sim 2.9\%$ is shown in Fig. 1 for the $v = 0$ toroidal plane. The safety factor profile (Fig. 2) has two widely separated $q = 2$ resonances at normalized minor radii $s/a \sim 0.25$ and 0.70. The blow-up in Fig. 1 shows the details of the innermost resonant islands. The $q = 5/2$ island chain is also clearly visible. The energy change compared with the initial nested surface equilibrium is $\delta W/W \sim -10^{-5}$. For this calculation, $N_s = 101$ (radial nodes), $M_u = 13$ (poloidal modes), and $M_v = 7$ (toroidal modes), corresponding to 27,573 independent variables and simultaneous nonlinear equations. The amount of CPU time required was less than 5 minutes on a single Pentium processor desktop computer. Present-day simulations for $S \sim 10^4$ run with $\sim 10^2$ radial points, whereas a full-volume ITER simulation with $S \sim 5 \times 10^8$ will require $\sim 10^4$ radial points to resolve narrow magnetic island structures. Angular resolution will also increase with an expected dense block structure approaching $(10^3–10^4)^2$ elements. Scalable solvers are being investigated that distribute the blocks on parallel processors. High-resolution ITER calculations with SIESTA will require petaflop scale computation on massively parallel platforms, which is planned in the near future.

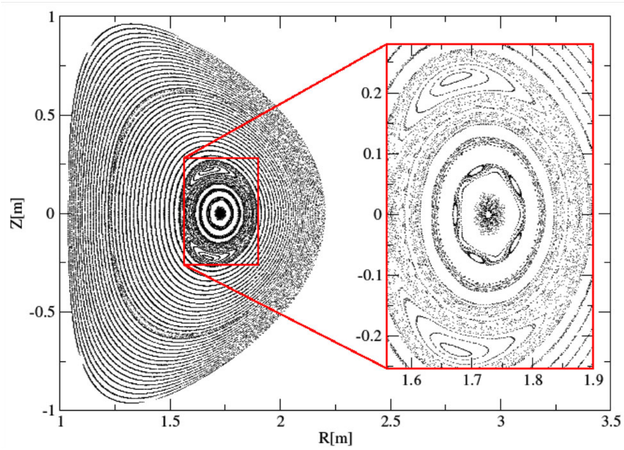

Fig. 1. Tokamak equilibrium with islands at the $q = 2$ resonant surfaces ($m = 2$, $n = -1$).

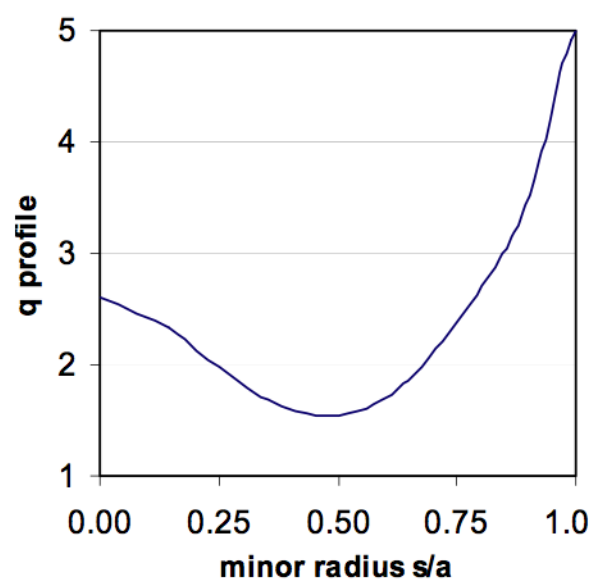

Fig. 2. Safety factor q for the example in Fig. 1.

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