stellarator news

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Multiregion relaxed MHD — A new approach to an old quandary

Three-dimensional (3D) MHD equilibria are still an outstanding theoretical and numerical challenge. The main reason is that current sheets are predicted to form at rational surfaces in general 3D equilibria with continuously nested flux surfaces [1]. Of course, the small but finite plasma resistivity allows these currents to diffuse and tear the magnetic field lines, thus forming magnetic islands around resonant rational surfaces. When islands are large enough to overlap, magnetic chaos emerges. Also, depending on the physical mechanisms at play, island self-healing can occur, in which case localized plasma currents can be sustained.

Independent of what mechanisms are at play in determining the saturated size of magnetic islands, the question of *how to compute the equilibrium magnetic field that is consistent with the established equilibrium pressure profile* is still under debate [2–5]. In fact, it is an outstanding challenge to compute 3D MHD equilibria —which generally consist of an intricate combination of flux surfaces, islands, and chaos—in a fast, robust, and verifiable fashion.

A recently developed theory based on a generalized energy principle, referred to as Multiregion, Relaxed MHD (MRxMHD), was developed [6] and elegantly bridges the gap between Taylor's relaxation theory and ideal MHD. MRxMHD allows for partial relaxation (which may lead to development of islands) and incorporates the possibility of non-smooth solutions (that describe current sheets).

The Stepped-Pressure Equilibrium Code (SPEC), a nonlinear implementation of MRxMHD, was developed in the last few years at PPPL [5]. SPEC has been benchmarked against VMEC in the axisymmetric case [7]; it is the first code ever tp compute 3D MHD equilibria with nested surfaces *and* the predicted singular current densities [8]; it was used to reproduce self-organized helical states in reversed field pinches [9]; and it has been used to study the response to resonant magnetic perturbations, with and without islands [5,10,11]. SPEC is clearly a candidate to compute 3D MHD stellarator equilibria with current sheets and magnetic islands, and has recently been brought from PPPL to IPP Greifswald in order to explore stellarator equilibria.

The SPEC code calculates MHD equilibria as extrema of the MRxMHD energy functional. In MRxMHD, the plasma is partitioned into a finite number, N, of nested volumes, V_ν, $\nu = 1, \ldots, N$, that undergo Taylor relaxation. These volumes are separated by $N - 1$ interfaces, I_ν, that are constrained to remain magnetic surfaces during the energy minimization process. The location and shape of these surfaces are a priori unknown and determined self-consistently by a force-balance condition. The MRxMHD equilibrium states satisfy:

$$\nabla \times \boldsymbol{B} = \mu_\nu \boldsymbol{B} \quad \text{in } V_\nu$$

$$\left[\left[p + \frac{B^2}{2}\right]\right] = 0 \quad \text{in } I_\nu$$

where $[[\ ]]_\nu$ is the jump across the νth interface and p is the plasma pressure, which is constant in each relaxed volume. While for $N = 1$ the theory trivially reduces to Taylor's theory, it has been shown that in the formal limit $N \to \infty$ ideal MHD is exactly retrieved [7].

In this issue . . .

As of now, SPEC is a fixed-boundary code and requires specification of the boundary in terms of the harmonics of its geometry. Akin to equilibrium codes, SPEC also needs specification of two profiles, e.g., the pressure in each relaxed volume, $p(\psi_v)$, and the rotational transform on either side of each interface, $\iota(\psi_v)$, in terms of the toroidal magnetic flux, ψ_v, enclosed in each volume. The SPEC code provides a solution for the magnetic field and the shape and location of the ideal interfaces.

As a first step towards predictive capability, SPEC has been verified for stellarator vacuum fields including islands. In fact, a vacuum field can be understood as a single Taylor state with $\mu = 0$. Thus SPEC can be used with $N = 1$, $\mu = 0$ (no current), and $p = 0$ (no pressure).

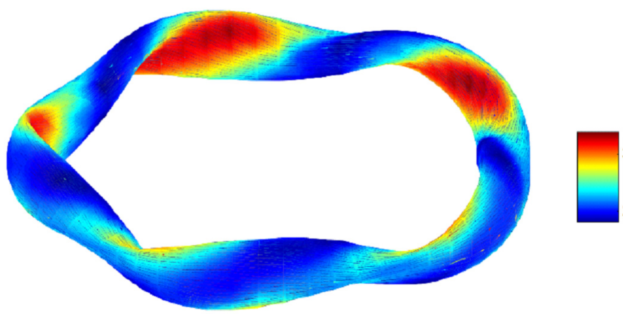

Fig. 1. Amplitude of the magnetic field on the W7-X boundary magnetic surface. Results obtained from SPEC.

As an example, Fig.1 shows the magnitude of the magnetic field obtained from SPEC on the outer surface of the Wendelstein 7-X (W7-X) limiter vacuum configuration, which includes a 5/6 island resonance. The solution has been exactly verified by showing that the volume-averaged error converges exponentially towards machine precision as the Fourier resolution is increased [12]. Also, a rigorous benchmark with Biot-Savart solutions has been successfully carried out (see, e.g., Fig. 2).

Finally, multivolume calculations, namely with $N > 1$, have also been verified, thus providing confidence that SPEC is correctly (i.e., with arbitrary accuracy) calculating MRxMHD equilbrium states.

This verification exercise has also motivated the search for faster algorithms that can maintain the current computation speed at high resolution in strongly shaped configurations such as W7-X.

The next steps to be undertaken are the calculation of stellarator equilibria with finite prescribed current and pressure in free boundary, which should provide insights into (1) the effect of bootstrap current on the formation of islands and (2) the equilibrium limit, which is related to the emergence of magnetically stochastic regions.

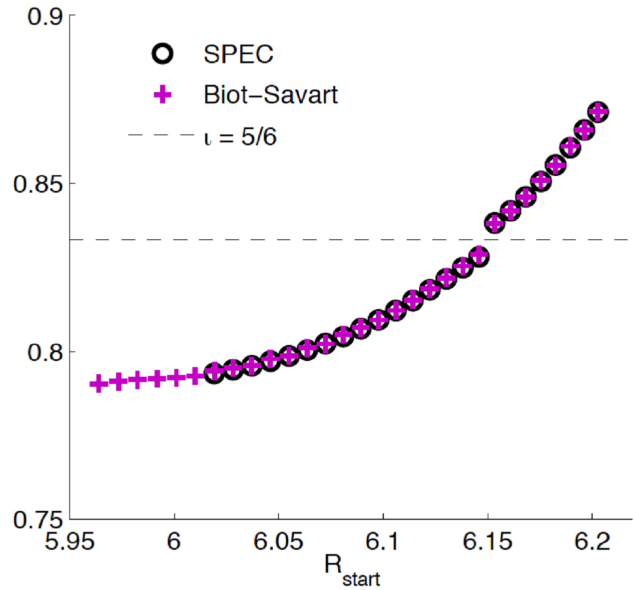

Fig. 2. Profile of the rotational transform obtained from field-line tracing on the SPEC and Biot-Savart solutions for the vacuum field in the W7-X limiter configuration.

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